Combinatorics (Permutations, Combinations with an example)

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1 Combinatorics

Combinatorics is an area of mathematics primarily concerned with counting, both as a means and an end in obtaining results, and certain properties of finite structures. The study of **permutations** and **combinations** is concerned with determining the number of different ways of arranging and selecting objects out of a given number of objects, without actually listing them.

Permutation is an **arrangement** of objects and **order matters**.

Combination is **selection** of objects from a collection and the **order of selection is immaterial**.

Fundamental Principle of Counting: Multiplication principle - If an event E can occur in ***m*** different ways and associated with each way of occurring of E, another event F can occur in ***n*** different ways, then the total number of occurrence of the two events in the given order is ***m* × *n***.

Addition principle – If an event E can occur in ***m*** ways and another event F can occur in ***n*** ways, and suppose that both can not occur together, then E or F can occur in ***m* + *n*** ways.

2 Notations used

Factorial Notation – Product of first ***n*** natural numbers is denoted by ***n!*** i.e. $n! = n.(n-1).(n-2). .... .3.2.1$

Permutation is arranging ***r*** objects out of ***n*** distinct objects

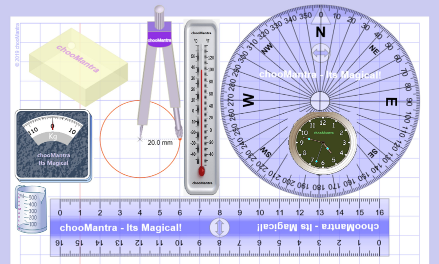
- when ***repetition is not allowed*** ${}_nP_r = n! / (n-r)!$
- when ***repetition is allowed*** ${}_nP'_r = n^r$

Combination is selecting ***r*** objects out of ***n*** distinct objects and order is immaterial ${}_nC_r = n! / [(n-r)!r!]$

Circular Permutation is arranging ***r*** objects in a circle out of ***n*** distinct objects

- when ***clockwise & anti-clockwise are different*** ${}_nP^c_r = {}_nP_r / r$
- when ***no distinction of direction*** is made ${}_nP^{c'}_r = {}_nP_r / 2r$

3 Part of chooMantra Maths Initiative



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4 Example Question

A security code is to be chosen using 6 of the following :
- the letters A, B, and C
- the numbers 2, 3 and 5
- the symbols * and $

None of the above may be used more than once. Find the number of different security codes that may be chosen if
(i) there are no restrictions
(ii) the security code starts with a letter and finishes with a symbol
(iii) the two symbols are next to each other in the security code.

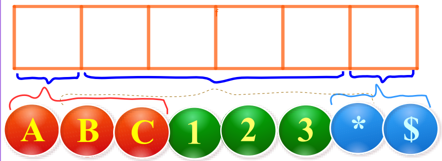
5 Solution Approach

Imagine 6 square boxes on a paper and 8 stickers as below. And based on restrictions, one needs to place the stickers (Stickers ensures none of them are used more than once)

6 Solution

(i) There are no restrictions. One can take any of 8 stickers and place it in each box. The 1st box can have any of 8 stickers, 2nd box any of 7 remaining stickers and so on. So the number of ways sticking can be done is **8x7x6x5x4x3=20160**
In other words, its the number of ways 6 stickers are chosen from 8 stickers and order of sticking matters, i.e. ${}_8P_6 = 8!/2! = 20160$

(ii) Security code starts with a letter & finishes with a symbol

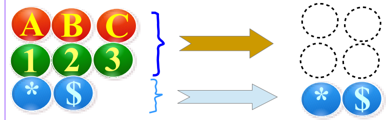


1st box can be filled by ${}_3P_1$ ways and 6th box can be filled by ${}_2P_1$ ways.
The remaining 4 boxes can be filled by remaining 6 stickers i.e. ${}_6P_4$.
So total ways = ${}_3P_1 x {}_6P_4 x {}_2P_1 = 3!/2! x 6!/2! x 2! = 2160$

7 Solution (continued)

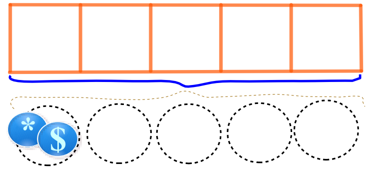
(iii) two symbols are next to each other in security code.

This can be done in following steps
1) First select 6 stickers. 2 symbol stickers and 4 other stickers from remaining stickers (order doesn't matter), i.e ${}_2C_2 x {}_6C_4$



8 Solution (continued)

2) Two symbol stickers are treated as joint sticker pair.



3) Now arrange the 5 stickers (including joint sticker pair and order matters) which is ${}_5P_5$ and symbol pair can arranged in ${}_2P_2$ ways. So the total no of ways is ${}_2C_2 x {}_6C_4 x {}_5P_5 x {}_2P_2 = 1x15x5!x2! = 15x120x2 = 3600$

9 Derangement, Distribution, etc.

Derangement is a permutation of n objects such that no element appears in its original position.

$$D(n) = n! [1-1/1!+1/2!-1/3!+ ... + (-1)^n/n!]$$

Distribution of n distinct objects into r groups $G_1, G_2, G_3, ... G_r$ containing $k_1, k_2, k_3, ... k_r$

- ***groups are distinct*** $= n!r!/(k_1! k_2! k_3! ... k_r!)$
- ***groups are identical*** $= n!/(k_1! k_2! k_3! ... k_r!)$

Special properties

${}_nP_n = n!$ ${}_nP_1 = n$ ${}_nP_r = n. {}_{(n-1)}P_{(r-1)} = {}_{(n-1)}P_r + r. {}_{(n-1)}P_{(r-1)} = {}_nC_r.r!$

${}_nC_r = {}_nC_{(n-r)}$ ${}_nC_r + {}_nC_{(r-1)} = {}_{(n+1)}C_r$ ${}_nC_0 + {}_nC_1 + ... {}_nC_n = 2^n$

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Apps, Skills, Gamification of education

https://choomantra.com/rajeevgm-notes/apps/

10 Special Cases

The **permutations** of ***n*** objects of which p_1 are of one kind, p_2 are of second kind, ... p_k are of k^{th} kind and the rest if any, are of different kinds is $n!/(p_1!p_2! ... p_k!)$

Restricted Permutation is arranging ***r*** objects out of ***n*** distinct objects if k particular objects are

- ***always included*** $= {}_{n-k}C_{r-k} . {}_rP_r$
- ***always excluded*** $= {}_{n-k}C_r . {}_rP_r$

Restricted Combination is selecting ***r*** objects out of ***n*** distinct objects if k particular objects are

- ***always included*** $= {}_{n-k}C_{r-k}$
- ***always excluded*** $= {}_{n-k}C_r$

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Play Country Capital Quiz on Alexa

http://bit.do/AlexaCountryCapitalQuiz

11 Alexa Country Capital Quiz

Country Capital Quiz by RajeevGM

Free to Enable

This is quiz on Countries and their capitals by @RajeevGM.

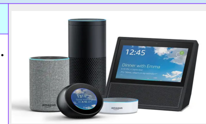
Play Country Capital Quiz, learn more about Geography and increase your General Knowledge.

12 Instructions to Enable and use Alexa Skill

Enable Skill

In Alexa app menu, select Skills. Search "Country Capital Quiz RajeevGM". Click on the skill displayed and click on "Enable Skill".

OR visit http://bit.do/AlexaCountryCapitalQuiz. And click on "Enable Skill". You will need to be logged into Alexa account



Use Skill

After enabling the skill, switch on Alexa enabled device. Once ready say "***Alexa, Launch Country Capital Quiz***"

Give a man a bowl of rice, you feed him for a day. Teach him farming, you feed him for life.